Rambaut's Mechanical Solution to Kepler's Problem

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Way of Introduction

Robert T. Gunther (1869-1940) was a man who knew the value of preservation and he keenly felt the loss or wanton destruction of any artifact having scientific value or historic interest. He railed against such extinctions whether ancient or modern.[1] The Museum of the History of Science (MHS) was founded as a consequence of Gunther's relentless lobbying in 1924, and it was the author's great pleasure during the summer of 2008 to visit Oxford in order to view an item that might reasonably be described as an extinct instrument, and one which could have easily been cast away and lost for ever (a brass parallel, if you will, of Tradescant's Dodo[1]). The instrument in question was designed by Arthur Alcock Rambaut (1859-1923) and its intended function was to solve Kepler's equation. A general history of the mechanical devices used or solving Kepler's equation has been presented in an earlier article[2], so our main objective here is to focus on Rambaut's instrument since his is the only known machine that has survived to the modern era.

Fig. 1 *Arthur Alcock Rambaut (1897–1923) at the time of his election to the Royal Society in 1900. Image courtesy of the RS.*

Background

Arthur Alcock Rambaut (Fig. 1) began his residency as fourth Radcliffe Observer, at the Radcliffe Observatory in Oxford, in 1897. He was to hold this post until the time of his death[3] on 14 October, 1923.

Shortly after Rambaut arrived at Oxford, the Radcliffe Trusties agreed to finance the installation of a new 24-inch aperture photographic survey telescope at the observatory. Built by the famous Grubb factory in Dublin, this telescope, complete with an 18-inch guide-scope, was installed at the Observatory towards the end of 1903 (Fig. 2).[4] Testing of the new telescope and its mount was conducted during most of 1904, and in 1905 a program of stellar parallax determinations via photographic plate measurements was initiated. The parallax survey work focused on Kapteyn's Selected Areas and formed part of a long running, but ultimately unsuccessful initiative to understand the structure of our Milky Way galaxy.

Parallel to the photographic survey work being conducted at the Radcliffe Observatory, Rambaut also made numerous observations of minor planets and comets. In addition, at various stages throughout his career, Rambaut published several theoretical papers concerning the orbits of binary star systems. It would have been in relation to determining the positions of minor Solar System bodies and the study of multiple star systems that Rambaut would have encountered the problem of solving Kepler's equation.

Judging from his collected publications it is clear that Rambaut was both a practical technician and a gifted theoretician. Indeed, on this latter point he obtained the Gold Medal for Mathematics and Mathematical Physics upon his graduation from Trinity College, Dublin in 1881. Evidence of Rambaut's interest in the design and construction of mathematical calculating devices can be found from the very earliest times of his post-graduate career. Writing in the Scientific Proceedings of the Royal Dublin Society in 1887, for example, Rambaut described a mechanical apparatus for enabling the conversion between astronomical and terrestrial coordinate systems.[5] This particular device (see Fig. 3) exemplifies Rambaut's geometrical approach to problem solving, the apparatus being composed of straightedges, pivoted rulers and semicircular protractor scales. Similar such components were also employed in the construction of his various mechanical devices for the solution of Kepler's equation.[2]

As an expert on the use of the transit circle, Rambaut set out in 1904 to perform a detailed analysis of the pivot errors displayed by the instrument housed at the Radcliffe Observatory. Rambaut's study followed in the wake of an earlier analysis by his predecessor, Edward James Stone, and related to a small but systematic discrepancy that existed between the right ascension determinations for stars observed with the Oxford, the Greenwich and the Cape Observatory transit circles. Stone had concluded that the problem did not lie with the Oxford transit circle, but Rambaut wanted to make sure of this fact for himself. The programme that Rambaut eventually followed required the design and construction of new testing and measuring apparatus, and the final analysis also required a highly refined mathematical investigation. Once again, we find Rambaut displaying the skills of an adept technician combined with those of a confident theoretician.

Rambaut's Mechanical Solution of Kepler's Equation

The machine developed by Rambaut, and now held in the Museum of the History of Science collection, was probably constructed *circa* 1905. The exact dates for its design and fabrication are not known, but Rambaut's paper in which the machine's operation is described was read at the 8 June, 1906 meeting of the Royal Astronomical

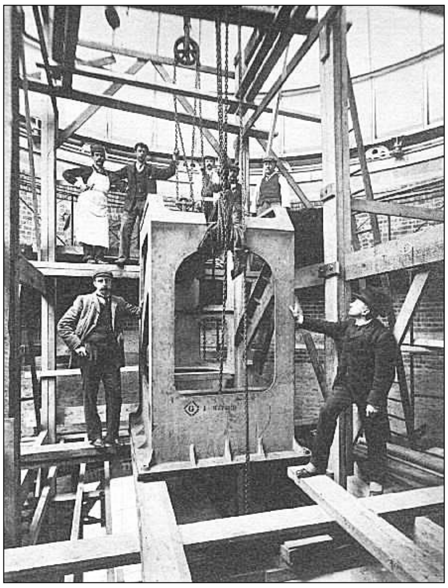

Fig. 2 *Installation of the new mount for the astrographic telescope at the Radcliffe Observatory. The photograph was taken circa 1902 and shows Rambaut, in hands-on fashion, to the left and Sir Howard Grubb to the right. Image courtesy of the University of London Observatory (http://www.ulo.ucl.ac.uk).*

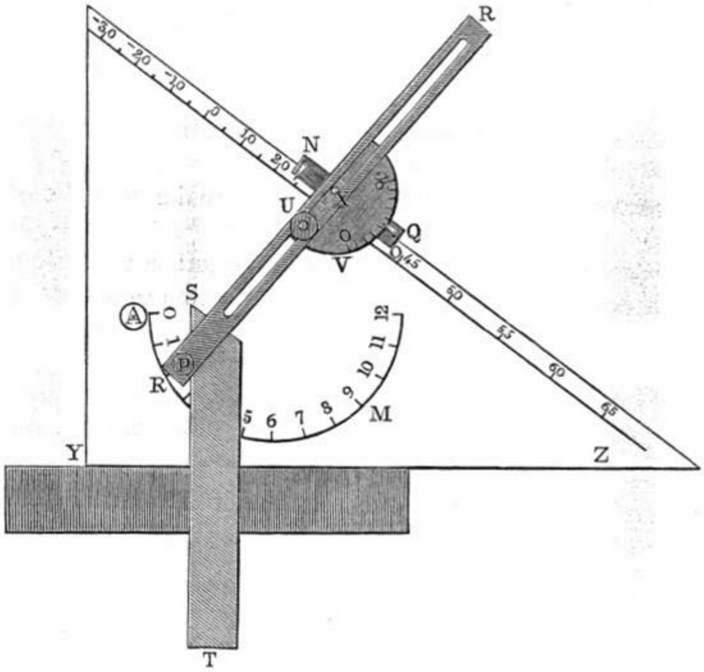

Fig. 3 *Rambaut's design for the mechanical conversion of hour angle and declination into altitude and azimuth. Rambaut comments that the devise is partly based upon a sundial design by the Jesuit priest François de Saint-Rigaud.*[6]

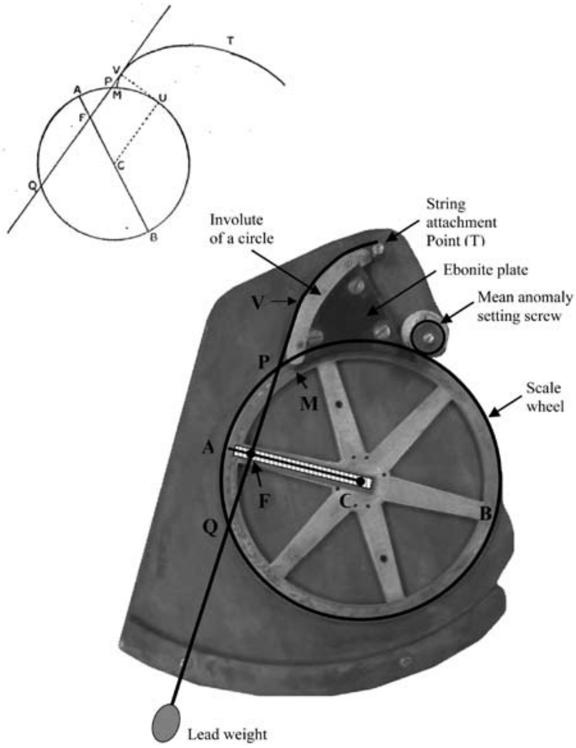

Fig. 4 *The solution geometry used by Rambaut to solve Kepler's Equation. The geometrical detail (upper left) is from ref. 8 - the corresponding points on the mechanical device are shown in the cut-out image.*

Society.[7] The underlying solution geometry applied by Rambaut[8] is shown in Fig. 4.

Kepler's equation $m = u - e \sin(u)$ specifies a relationship between the mean anomaly m, the orbital eccentricity e and the eccentric anomaly u. In general it is the eccentric anomaly that is to be solved for given an observational determined estimate of the orbital eccentricity and a specified mean anomaly. Since there is no closed analytic solution to Kepler's equation graphical or iterative numerical schemes are generally used[2] to find the required solution for u.

With respect to Fig. 4 (as reproduced here), Rambaut explains[8], "let us take a circle, AMB, …, with centre C and any radius AC, and lay off the arc AM corresponding to the angle m. From M draw [a] portion of the involute MVT,… Divide AC at F so that FC/AC = e [the eccentricity]. From F draw a tangent to the involute at V cutting the circle in P and Q. Then the angle AFV = u [the eccentric anomaly to be found]….". Rambaut then goes on to prove that angle ACU = AFV = u, and that angle ACU = (AP + BQ) / 2. The working principles of the machine are now determined. For a given value of the mean anomaly m and a predetermined expression for the eccentricity e, a thin string VPFQ is used to determine the angles AP and BQ, and from the average of these angles the eccentric anomaly u can be calculated. Once the eccentric anomaly is known then the elliptical coordinates corresponding to the distance from the primary focus, and the true anomaly can be evaluated. These latter two quantities indicate the position of the object (star, asteroid, comet or planet) in its elliptical orbit at the time corresponding to the mean anomaly m.

A Description of Rambaut's Instrument

The Radcliffe Observatory was founded at Oxford University in 1772 and saw continuous activity until the end of 1934, when operations were moved to Pretoria in South Africa. Initially, the holder of the Savilian Chair of Astronomy was responsible for observatory operations, but in 1839, when the theoretician George H. S. Johnson (1808-1881) was appointed to the post, it was decided to create the new role of Radcliffe Observer. It thereafter became the latter's duty to oversee the observational programs carried out at the Observatory. Harold Knox-Shaw (1885-1970), Rambaut's successor, oversaw the final years of operations at Oxford, and in 1935 a number of smaller instruments and associated astronomical ephemera were transferred from the Radcliffe Observatory to the collection of the Museum of the History of Science.[4] Rambaut's mechanical solution for Kepler's equation was one of the instruments transferred (MHS inventory number 11031).

Rambaut's device is shown in Figs 5a and 5b. The base plate of 1/8th-inch brass plate is essentially a quadrilateral with a rounded base of length 23 cm, and three straight sides of approximate length 23, 11 (at the top) and 23 cm. Indeed, the base plate is a solid piece of material and while the physical mode of operation is not explained by Rambaut, it is clear that the device was intended to rest upon a tabletop, rather than being hand-held when in operation. Indeed, three, ¾-inch long cylindrical brass legs have been attached to the underside of the base plate to raise it above the supporting surface, and through which a drop space is provided for the lead weight, attached to the measuring string, to fall (see Fig. 5b). The lead weight was employed in order to keep the string taught and to thereby reduce the error in both reading and setting the scale-wheel. The scale-wheel is 6-inches in diameter and attached to the base plate so that a small thumbscrew can be used to control the degree of its rotation. The scale-wheel is made of silvered brass and has been divided in half-degree increments. Rambaut indicates in his explanatory article that the scale-wheel has been adapted from "a small setting circle taken from the old Jones Meridian Circle".[9] Screwed to one of the radial arms of the scale-wheel (corresponding to CA in Fig. 4) is an eccentricity scale plate. The silvered eccentricity scale plate is divided from 0 to 1 in units of 1/10th. The zero point for the eccentricity

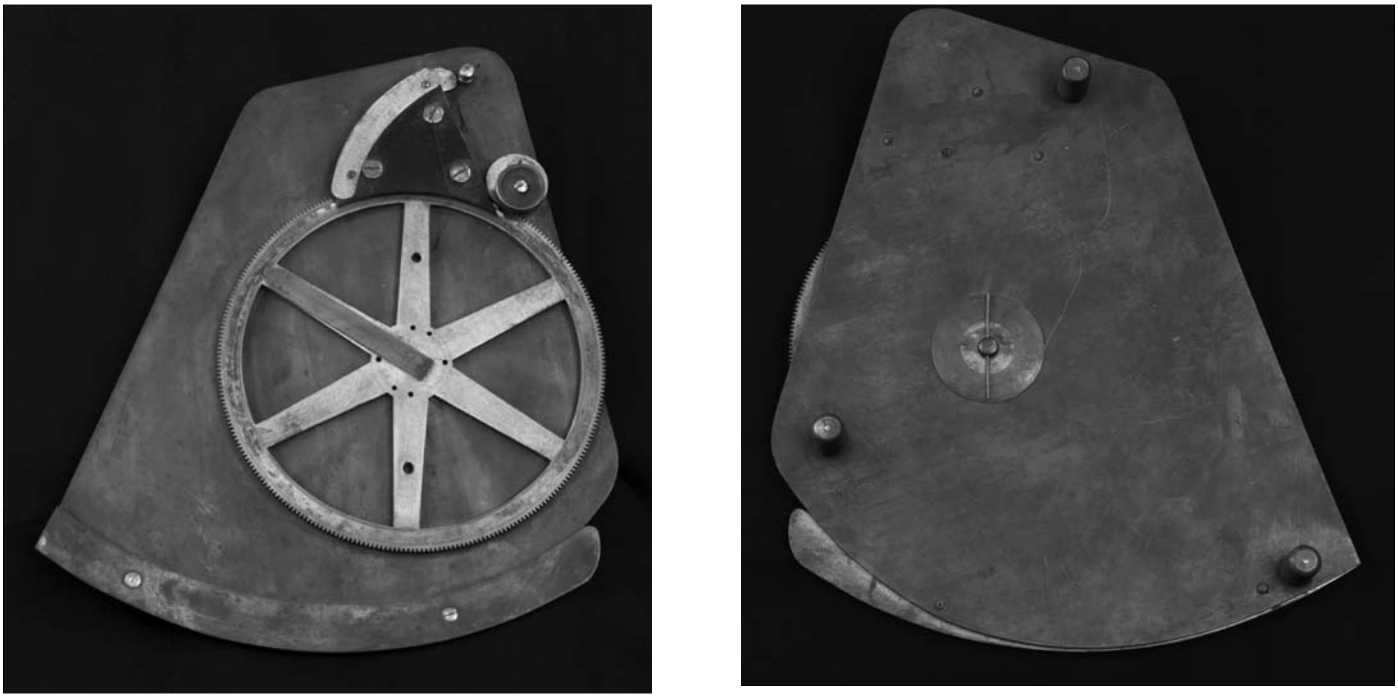

Fig. *5 (a) Rambaut's mechanical device for finding solutions to Kepler's Equation –* fecit *Henry Minn* c.*1905/6. (b) Reverse side of Rambaut's instrument. Museum of the History of Science inventory number 11031. Photographs by Richard Rowley.*

scale is located at the center of the scale-wheel (corresponding to point C in Fig. 4) while the limiting eccentricity reading of unity is located on its outer rim.

The key solution component of Rambaut's device is the arc corresponding to the involute of a circle (arc MVT in Fig. 4).An examination of the MHS device reveals that the involute plate has been cut from a roughly triangular piece of $1/8^{th}$-inch thick ebonite. The arc of the involute subtends an angle of about 70 degrees, and it has evidently been well crafted.

In his 1906 *Monthly Notices* paper, Rambaut comments that "the instrument has been constructed for me by a skilful watchmaker – Mr. H. Minn, of Oxford".[8] Henry Minn (1870–1961) traded, as his father had before him, as a watchmaker until 1914, when he joined Oxford University Press as an engineer.[10] Later in life he became a familiar figure at the Bodleian Library, and was known for his photographic studies depicting the streets of Oxford. Minn was also engaged by Rambaut in the construction of several components for his 1904 investigation of the pivot error in the Oxford transit circle. It is not presently clear how the association between Minn and Rambaut came about, but it is known that Minn was interested in early clocks and scientific instruments – topics that were also of great interest to Rambaut. It may have been this common interest that facilitated their initial meeting.

Understandably, Ranbaut's device is now looking a little tarnished, having not been used for the best part of the last 80 years, and the silvered-brass scales are faded and difficult to read. This being said, the device is still fully functional and the only repair that would be required to begin the performance of actual calculations is the attachment of a new string for the lead weight.

Acknowledgments

It is my great pleasure to thank and acknowledge Lucy Blaxland and Tony Simcock of the Museum of the History of Science, Oxford for their generous help in finding material relating to Rambaut and for their very pleasant rapport during my visit to the Museum in 2008.

Notes and References

1. A. V Simcock, ed., *Robert T. Gunther and the Old Ashmolean* (Oxford: Museum of the History of Science, 1985). One of the great treasures within the Tradescant collection was a complete specimen of the now extinct Dodo. Acquired by Elias Ashmole upon Tradescant's death in 1662, the collection formed the foundation for the [Old] Ashmolian Museum at Oxford – originally located, in fact, in the same building now occupied by the MHS. This unique item, currently part of the Oxford University Museum collection, is the oldest known specimen of a Dodo and the only specimen in the world with preserved soft tissue. Only the head and one foot of the original Dodo survive, however, since the original bird was ordered incinerated in 1755 as a result of its neglected and deteriorating state. To Gunther the destruction of this now iconic "more extinct than almost any other extinct animal" [Simcock, p. 81] specimen, and its loss to future researchers was a rallying cry for the conservation and preservation of all items of scientific interest. An illustrated broadsheet concerning the Oxford Dodo can be found at: http://www.oum.ox.ac.uk/learning/pdfs/dodo.pdf.

2. M. Beech. 'Solving Kepler's Problem Mechanically', *Bulletin of the Scientific Instrument Society*, No. 100 (2009), pp. 6-12.

3. Third son of the Reverend Edmund F. Rambaut, Arthur Alcock Rambaut was born at Waterford, Ireland on 21 September, 1859. He was educated at Arlington House, Portarlington, the Royal School, Armagh and Trinity College, Dublin. He graduated from Trinity as senior moderator and Gold Medallist in mathematics and mathematical physics in 1881. For just a few months following his graduation Rambaut held the post of Senior Science Master at the Royal School, his old *Alma Marta*, in Armagh, but soon joined the Dunsink Observatory as an assistant astronomer working under the directorship of Robert Ball. He married Emily Langford in 1883 and over the ensuing years fathered three sons, all of which were born at the assistant's house at Dunsink. Rambaut was elected a Fellow of the Royal Society in 1900, and a Fellow of the Royal Astronomical Society in 1893, serving on the latter societies Council from 1901 to 1906 and from 1914 to 1918. In 1892 Robert Ball moved to Cambridge University and Rambaut was appointed

Andrews Professor of Astronomy at Trinity, and Royal Astronomer of Ireland. With the death of E. J. Stone, Rambaut took the opportunity to apply for the position of Radcliffe Observer on 24 June, 1897 with letters of support from Ball, mathematician George Stokes, physicists George Stoney, and George FitzGerald and astronomer William Huggins. Upon arriving at Oxford in 1897, Rambaut became a member of Queen's College. In 1922 Rambaut suffered a serious collapse of his health and retired to the Moorcroft Nursing Home in Uxbridge. His death on Sunday 14 October, 1923 was recorded in the *Times of London* newspaper (for October 16th), and following a Service at the Chapel of Queens College on October 17th Rambaut was interned at Holywell. Obituary notices for Rambaut were published in *The Observatory*, **46** (1923), pp. 326-327, *The Monthly Notices of the Royal Astronomical Society*, **84** (1923), pp. 220-221, and the *Proceedings of the Royal Society*, **106** (1924), pp. ix-xii. P. A. Wayman, in the *Oxford Dictionary of National Biography*, also provides a brief account of Rambaut's life and carrier. Likewise, see P. A. Wayman, 'The Andrews' Professors of astronomy and Dunsink Observatory, 1765–1985', *The Irish Astronomical Journal*, **17**(3) (1986), pp. 165–184.

4. When it was decided to move the Radcliffe Observatory operations to Pretoria, South Africa, the 18/24-inch telescope system was offered, in 1935, to the University of London Observatory. The telescope, complete with its German equatorial mount and rising observatory floor were accordingly transported to the Mill Hill Observatory. The new site was officially opened by the Astronomer Royal, Sir Howard Spencer Jones on 1 July, 1938. The telescope was refurbished and cleaned in 1994 and is still in use to this very day. Further details can be found at http//www.ulo.ucl.ac.uk/.

5. A. A. Rambaut, 'On a mechanical method of converting hour-angle and declination into altitude and azimuth and of solving other problems in spherical trigonometry', *Scientific Proceedings of the Royal Dublin Society*, **5** (8) (1887), pp. 642–645. Rambaut published some 23 years on from his RDS paper a lengthy theoretical article on standard coordinate transforms in the *Monthly Notices of the Royal Astronomical Society*, **70** (1910), pp. 655-672.

6. See for example, F. A. Stebbins. A Mediaeval Portable Sundial. *Journal of the Royal Astronomical Society of Canada*, **55** (2) (1961), pp. 49-56.

7. Listed in *The Observatory*, **29** (1906), p. 275.

8. A. A. Rambaut. A simple method of obtaining an approximate solution of Kepler's Equation', *Monthly Notices of the Royal Astronomical Society*, **66** (1906), pp. 519-521.

9. R.T. Gunther, *Early Science in Oxford*, Vol. 11 (1923), p.326 indicates that a 6-foot diameter Meridian circle, with a 4.1-inch objective telescope, constructed by Thomas Jones (1775–1852) of Charing Cross, London was installed at the Radcliffe Observatory in 1836 and was apparently in use until 1861. The circle was predominantly used by Manuel Johnson who is perhaps better known in the modern era for his southern hemisphere catalogue of star positions obtained from the island of Saint Helena. Gunther suggests that the instrument was designed by Astronomer Royal (1806–1835) John Pond.

10. The Bodleian Library Record VII: 2, July 1963. I am very much indebted to Tony Simcock for providing this reference concerning Minn.

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Current and Future Events

Details of future events, meetings, exhibitions, etc. should be sent to the Editor. For up-to date information of Society's events, see the SIS website, www.sis.org.uk

Until 11 December 2009, Harvard, Cambridge, Mass, USA
A new exhibit at Harvard's Collection of Historical Scientific Instruments: *Patent Republic: Materialities of Intellectual Property in 19th-Century America*, curated by Jean-Francois Gauvin. Researched and designed by the students in Prof. Mario Biagioli's course 'Intellectual Property in Science': Kristina Bergquist, Jeffrey Clinton Holder, Pierce Tria, William Ball, Wintha Kelati, Jasmine Rencher, Diana MacLean, Michael Wolfe, David Haber, Kiran Reddy Pendri, Jeffrey James
Blair, Colin Flood, and Danielle Charlap.

This exhibit which retraces more than 50 years of patent-model making in the United States, provides clues to the epistemic nature of models and their role in both technological innovation and the history of intellectual property law. It can be visited on weekdays only between 9 am and 5 pm.

Friday 19 February 2010, London, England
Visit to the Royal College of Physicians' Headquarters and museum collections from 2.30 to 5.30 pm - to be confirmed.

Friday 7 May 2010, Greenwich, London, England
A Sense of Direction - New Researches in the History of Navigation, symposium held at the National Maritime Museum in Greenwich.
Further details at http://www.nmm.ac.uk/researchers/conferences-and-seminars/

Monday10 May - Friday 14 May 2010, Germany: Overseas Study Conference to Frankfurt and Kassel
Provisional Programme
Frankfurt 11 May: Morning: handling session of instruments with Reinhard Glasemann, who in 1999 published a descriptive catalogue in German of the museum's rich collection of globes, sun-dials and astronomical instruments. Afternoon: Institute for the History of Arabic-Islamic Sciences, which has built up an amazing collection of replica instruments, see http://web.uni-frankfurt.de/fb13/igaiw/index.html click (German only).

Wednesday 12 May: Coach to Kassel. En route visit to the Liebig Museum in Giessen, 'the mother of all chemical laboratories', see http://www.liebig-museum.de/ (German only) and the picturesque old university town Marburg, where we will see the 19th century observatory and the university collection of physics instruments see http://www.uni-marburg.de/einrichtungen/sammlung (German only).

Thursday 13 May: Instruments and clocks in the museum 'Astronomisch-Physikalisches Kabinett' http://www.museum-kassel.de/index_navi.php?parent=1035 (German only) and other attractions in Kassel.

Friday 14 May: Instruments at the University of Goettingen, see http://www.uni-goettingen.de/de/47114.html and more attractions in Kassel. Conference Dinner.

We invite expressions of interest. The Society will make arrangements for accommodation (2 nights Frankfurt, 2 nights Kassel), group meals and coach hire for transport within Germany. Delegates to make own travel arrangements to and from Frankfurt. Please see the flyer in this *Bulletin*.

A Saturday in July 2010, Cambridge, England
The Society's Annual General Meeting and a day of postgraduate talks will be held in July in Cambridge. Date to be announced.

October 2010, English Provincial Visit
Details to be confirmed.

Friday 19 November 2010, London, England
Eighteenth Annual Invitation Lecture will be held at the Society of Antiquaries
Precise title to be confirmed
Dr Liba Taub of the Whipple Museum, University of Cambridge will speak on preserving 19th and 20th century artefacts. Doors open at 5.30 pm to start at 6 pm.